Stability of light beams in nonlinear antiwaveguides

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We consider the standard models of the nonlinear light-guiding systems in the form of a core sheathed by a cladding with a different refractive index, the Kerr coefficient being the same in both media. Recently, it has been demonstrated that this system may support a light beam localized near the core not only in the case of the usual waveguide configuration, when the core is optically denser than the cladding, but also in the opposite case (the *antiwaveguide*). In this work, we compute the effective Hamiltonian of the localized beam (normalized to the number of quanta) versus the refractive index difference. We demonstrate that, while this dependence is trivial in the waveguide case, for the antiwaveguide it reveals nontrivial minima at special values of the parameters. These minima may be a strong argument in favor of stability of the corresponding antiwaveguide states. We compare the loci of the minima with the possible stability regions predicted recently by means of another heuristic criterion. The comparison yields an additional argument in favor of the stability.

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It is well known that light can propagate in nonlinear waveguides, a silica fiber being a typical example [1]. In a general case, the waveguide may be regarded as a core of an optically dense material sheathed by a less dense cladding. However, it has been recently demonstrated [2] that an *antiwaveguide* cylindrical configuration, in the form of a less dense core in a denser cladding, may also support a light beam localized near the axis. It is necessary to note that, for planar nonlinear optical systems, the antiwaveguide configurations were considered in a number of works [3–6].

Evidently, a crucial issue is the stability of these beams [5–7]. In Refs. [2,7], curves of the eigenvalues (the field amplitude in the center of the core) of the corresponding nonlinear boundary problem versus the refractive index difference between the core and the cladding have been obtained numerically. On those curves, nearly vertical segments were found in certain parametric regions. Heuristic arguments were given in favor of stability of regions corresponding to the vertical segments.

As a full stability analysis is very hard, in this work we aim to develop a simplified approach to the stability problem based upon energy arguments. Using the antiwaveguide states found numerically in Refs. [2,7], we compute the values of the corresponding effective Hamiltonian, normalized to the number of quanta of the beam. The main finding is that the curves of the normalized value of the Hamiltonian versus the control parameter (refractive index difference) demonstrate nontrivial local minima in certain regions both for the cylindrical and planar geometries. It is necessary to emphasize that the minima are found only for the antiwaveguides, and never for the usual waveguide states. Intriguingly, the location of the minima practically exactly coincides, for both geometries, with location of one of the abovementioned vertical segments in the dependences of the eigenvalue versus the control parameter. Thus the corresponding parametric regions have really good chances to support stable planar or cylindrical nonlinear light channels. Comparison with the results of Refs. [2,7] shows also that points at which the eigenvalue dependences have the vertical segments *always* correspond to some critical points (which, however, are not necessarily minima) in the dependence of the Hamiltonian upon the control parameter. Thus the presence of the vertical segments seems to be only necessary, but not sufficient, for the stability. It is also relevant to emphasize that, since the Hamiltonian minima are discovered, as a matter of fact, in narrow parametric regions, the results obtained in this work may give practically important information for the search of antiwaveguide configurations producing stable light beams.

We start our analysis with the general nonlinear Schrödinger equation governing distributions of the electromagnetic field envelope $u(z,x)$ along the longitudinal coordinate z and the transverse coordinate x, which can be derived from Maxwell's equations for a Kerr focusing medium and inhomogeneous refractive index [8]:

$$iu_z + \frac{1}{2}u_{xx} + \frac{1}{2}(N-1)x^{-1}u_x + |u|^2u - U(x)\,u = 0\ . \qquad (1)$$

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Here x is the transverse Cartesian coordinate for the planar ($N = 1$) configuration, or the transverse radial coordinate for the axisymmetric cylindrical ($N = 2$) configuration. The "potential" $U(x)$ is defined as follows [2]: $U \equiv 0$ for $x > x_c$, and $U = U_0$ for $x < x_c$, where x_c is the core half thickness or the core radius for $N = 1$ and $N = 2$, respectively. The constant U_0 is proportional to the refractive index difference between the cladding and the core. Negative and positive values of U_0 correspond to the ordinary nonlinear waveguides and antiwaveguides, respectively.

The effective Hamiltonian corresponding to Eq. (1) is

$$H = \int (2\pi x)^{N-1} dx \left(\frac{1}{2}|u_x|^2 - \frac{1}{2}|u|^4 + U(x)|u|^2 \right), \tag{2}$$

where the integration is from $-\infty$ to $+\infty$ in the case of $N = 1$, and from 0 to $+\infty$ in the case of $N = 2$. We look for a solution to Eq. (1) in the form

$$u(z,x) = \sqrt{\beta_0} R(r) \exp(i\beta_0 z), \tag{3}$$

where $r \equiv x\sqrt{2\beta_0}$, β_0 is the propagation constant of the electric-field envelope (unlike the propagation constant of the carrier wave β from Ref. [2]), and the multiplier $\sqrt{\beta_0}$ in front of $R(r)$ is introduced for convenience. Notice that, due to the presence of this multiplier, the renormalized radial coordinate r depends too on the propagation constant. Insertion of Eq. (3) into Eq. (1) leads to the following equation for the real amplitude $R(r)$:

$$\frac{d^2R}{dr^2} + (N-1)r^{-1}\frac{dR}{dr} = [1 + \beta_0^{-1}U(r)]R - R^3. \tag{4}$$

We limit ourselves to solutions that correspond to the fundamental mode of the ordinary linear waveguide. That is, we require bounded, square integrable solutions for Eq. (4) which are non-negative for all values of r. Furthermore, they must be symmetrical, i.e., $R(-r) = R(r)$ and $R'(0) = 0$, and both $R(r)$ and $R'(r)$ must be continuous at the core-clad interface at $r = r_c$, where $r_c \equiv x_c\sqrt{2\beta_0}$. The constant $A \equiv -U_0/\beta_0$ plays the role of the control parameter as in Ref. [2], while $R_0 \equiv R(0)$ will be regarded as an eigenvalue of Eq. (4).

Numerical solutions of Eq. (4) were reported in Ref. [2] for $N = 2$, and in Ref. [7] for $N = 1$. Note that, at certain values of the parameters, the solution satisfying this condition is not unique; it is also interesting to note that, in some cases, the ground-state solution can be a nonmonotonic function of r (i.e., it may have local maxima and minima at $r \neq 0$). Once the solution is found, it can be substituted into Eq. (2) to calculate the corresponding value of the Hamiltonian. Integrating Eq. (2) by parts, and making use of Eq. (4), one can cast the Hamiltonian into the following final form:

$$H = \int (2\beta_0)^{-N/2} (2\pi r)^{N-1} dr\, \beta_0^2 \left(\frac{1}{2}R^4 - R^2 \right). \tag{5}$$

Knowing the value of the Hamiltonian corresponding to a given solution may assist in establishing stability of the solution (see, e.g., Ref. [9]). First, if the Hamiltonian is negative, this at least guarantees that the waveguide or antiwaveguide state cannot simply decay into quasilinear (small-amplitude) waves, as the Hamiltonian is a constant of motion, and, as it follows from Eq. (2), the Hamiltonian of the quasilinear waves is always positive. Indeed, for small amplitude, one can neglect the fourth order term in expression (2), so that it becomes positively definite [alternatively, one can use for the Hamiltonian an expression of the type (5) in which, however, it is necessary to take into account that the propagation constant for the quasilinear waves has a sign opposite to that for the waveguide or antiwaveguide mode]. Moreover, since any transient process related to emission of the quasilinear waves may only decrease the value of the Hamiltonian of the remaining state, it is natural to expect (although this is an assertion based on physical intuition rather than a rigorous theorem) that a local minimum of the Hamiltonian, if any, should give rise to a stable state. On the other hand, if the values of the Hamiltonian are bounded from below, this may be viewed as a guarantee against another sort of instability, viz., collapse [9]. The results presented below clearly show that the Hamiltonian is indeed bounded from below, at least within the class of all the (anti)waveguide modes.

In what follows below, we will display results of numerical computation of the integral (5) with the eigenfunctions borrowed from Refs. [2,7]. It is relevant to normalize the Hamiltonian, dividing it by another constant of motion, the "number of quanta"

$$Q \equiv \int (2\pi x)^{N-1} dx\, |u(x)|^2.$$

In Figs. 1 and 2, we display the normalized Hamiltonian $h \equiv H/(Q\beta_0)$ versus the control parameter A for some certain values of r_c. In the definition of h, the multiplier β_0^{-1} was introduced for some technical reasons related to a procedure of plotting the dependences; actually, at all the spots of interest, β_0 will be practically a constant, so that this factor does not play any role. Anyway, following the definitions of the scaled quantities r, Q, and h, it is easy to interpret the results displayed below in terms of the physical parameters of the system. In most cases, the computations yield rather trivial dependence with no local minima of h. However, for values of r_c close to 2.3, we were able to find a local minimum of $h(A)$ located at some negative value of A, i.e., just for the antiwaveguide (see top portions of Figs. 1 and 2). In the lower portions of Figs. 1 and 2, we display the eigenvalue R_0 versus A (see Ref. [2]). One notes that the local minima of $h(A)$ exactly coincide with the values of A corresponding to vertical segments of $R_0(A)$. In Refs. [2,3] it was argued heuristically that the vertical segments might correspond to stable configurations. Note that near the vertical segments the parameters A and β_0 change insignificantly. The results shown in Figs. 1 and 2 give an additional strong argument in favor of stability of the configurations corresponding to these values of A.

The purport of the results obtained is that they indi-

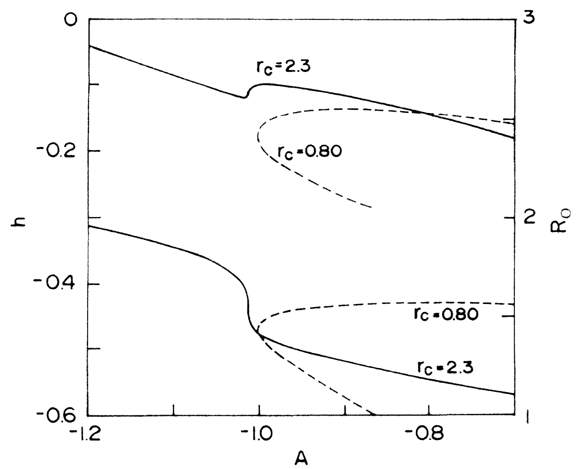

FIG. 1. The normalized Hamiltonian h vs the control parameter A in the planar system ($N = 1$) for the core's thickness $r_c = 0.80$ (broken line) and $r_c = 2.3$ (solid line). Beneath it, the eigenvalue R_0 is shown vs A as per Ref. [3].

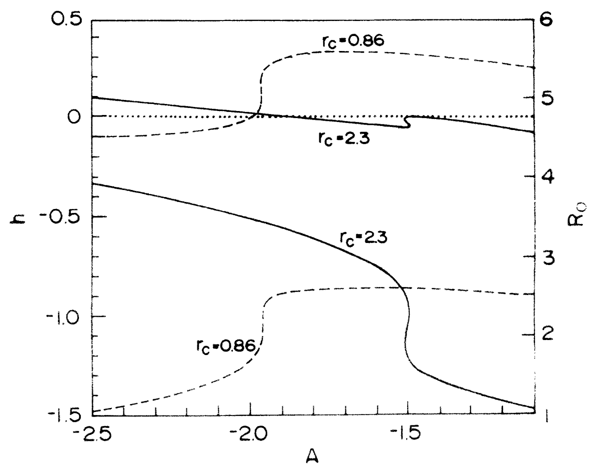

FIG. 2. The same as in Fig. 1 in the cylindrical system ($N = 2$) for the core's radius $r_c = 2.3$ and $r_c = 0.86$. The plot of R_0 vs A is taken from Ref. [2].

cate how to choose very special values of the parameters r_c and A of the nonlinear system, at which the system may support stable antiwaveguide states. For example, for the planar antiwaveguide, the value $A = -1$ seems eligible. It is noteworthy that we have *never* been able to find a local minimum of the Hamiltonian for positive A, i.e., for the ordinary nonlinear waveguide. Note also that vertical segments of $R_0(A)$ were found (see Refs. [2,7]) not only near the Hamiltonian minima of the present work but in other regions of the parameter A, as well. In each case, the vertical segments are correlated with certain peculiarities in the corresponding curves of $h(A)$. As an example, in Figs. 1 and 2 we display the results obtained for $r_c = 0.80$ ($N = 1$) and for $r_c = 0.86$ ($N = 2$) (the presence of the turning point in the plots shown in Fig. 1 means that there is no solution to the left of it, and there are two different solution branches on the right side). Comparing the upper and lower parts of these figures, we note that the vertical segments are located at values of A at which some critical points exist with a very steep variation of $h(A)$ but not local minima. However, there is no reason to assume that these critical points (unlike the local minima) should give rise to stable states. This observation implies that the presence of the vertical segments may be only a necessary, but not sufficient, stability condition.

In conclusion, we consider an example for pulse parameters applicable to nonlinear silica-fiber antiwaveguides [2]. Assuming pulse energy of 4.4×10^{-5} J, pulse duration of 10 ps at carrier wavelength of 1 μm, and Kerr coefficient of 1.83×10^{-22} m^2/V^2 [10], with refractive index difference of 0.01 between the core and cladding, one obtains $r_c \cong 2.3$ and $A \cong -1.5$ which corresponds to the Hamiltonian minimum in Fig. 2. Note that the optical energy density approximately equals the density necessary for self-focusing in a homogeneous material.

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